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Electromagnetic Sub-wavelength Imaging Using The Basis Matrix Method In Conjunction with Singular Value Decomposition (SVD) Algorithm

(1) (2)Xiang Gu, (1)Raj Mittra, and (2)Yunhua Zhang

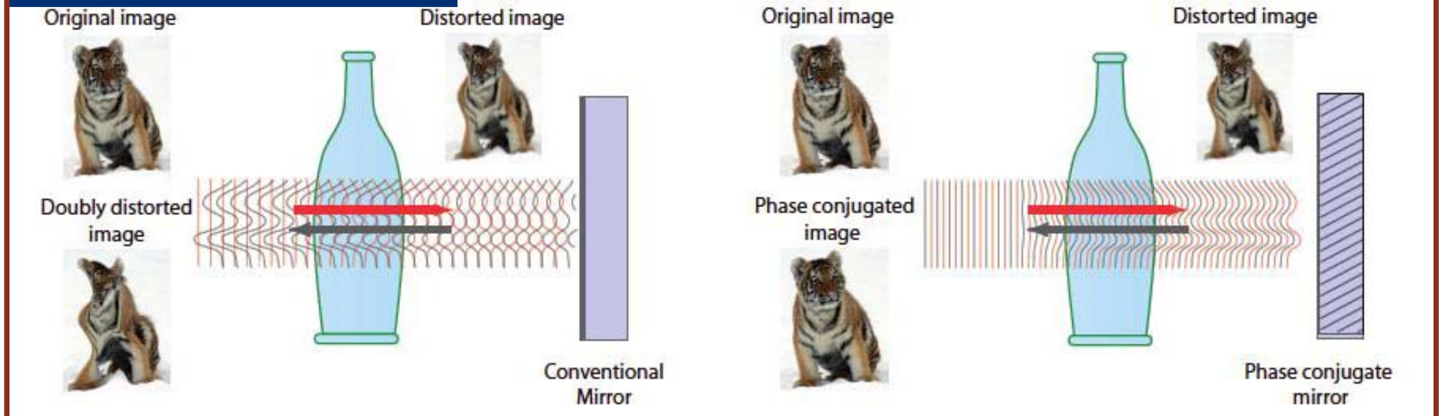
rajmittra@ieee.org

(1) EMC Lab, Department of Electrical Engineering, The Pennsylvania State University, State College, PA, USA, 16802
(2) Key Laboratory of Microwave Remote Sensing, Centre for Space Science and Applied Research, Chinese Academy of Sciences, Beijing, 100190, China

Outline

- Introduction
- Electromagnetic sub-wavelength imaging model
- Phase Conjugation (PC)
- Basis matrix method with Singular Value Decomposition (SVD)
- Numerical results

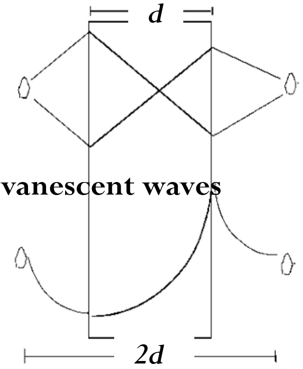
Phase Conjugation Concept


Original image
Distorted image
Doubly distorted image
Conventional Mirror
Original image
Distorted image
Phase conjugated image
Phase conjugate mirror

*The Changhuei Yang Research Group at Biophotonics Laboratory, California Institute of Technology

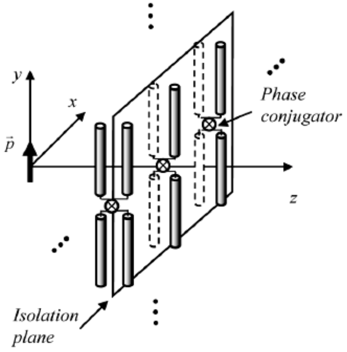
Introduction

Propagating waves


d
Evanescent waves
2d

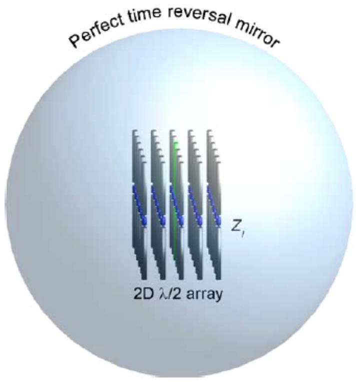
Focusing of propagating (upper) and evanescent (lower) waves in the Pendry superlens.

* A. Neice, "Methods and limitations of subwavelength imaging," *Adv. Imag. Elect. Phys.*, vol. 163, no. 18, pp. 117-140, 2010.


y
x
$\vec{p}$
z
Phase conjugator
Isolation plane

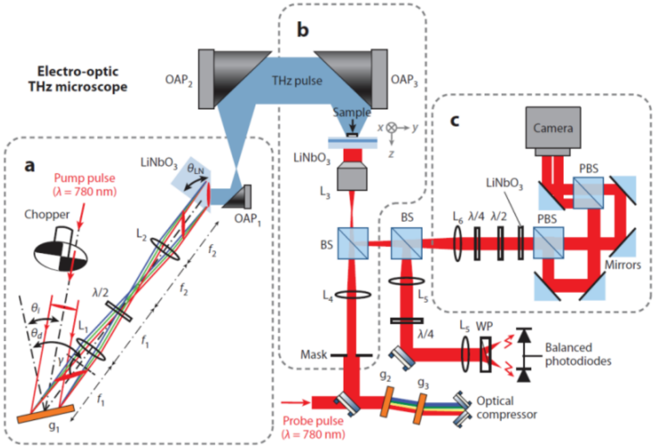
Double sided PC lens

* O. Malyuskin, and V. Fusco, "Far field subwavelength source resolution using phase conjugating lens assisted with evanescent-to-propagating spectrum conversion," *IEEE Trans. Antennas Propag.*, vol. 58, no. 2, pp. 459–468, 2010.


Perfect time reversal mirror
Z_i
2D λ/2 array

5 by 5 array of $\lambda/2$ dipoles inside a "perfect" time reversal mirror

* J. de Rosny, G. Lerosey, and M. Fink, "Theory of Electromagnetic Time-Reversal Mirrors," *IEEE Trans. Antennas Propag.*, vol. 58, no. 10, pp. 3139–3149, 2010.


Electro-optic THz microscope
a
b
c
OAP$_2$
THz pulse
OAP$_3$
OAP$_1$
Sample
LiNbO$_3$
θ_{LN}
Pump pulse (λ = 780 nm)
Chopper
L$_2$
L$_1$
L$_3$
L$_4$
L$_5$
L$_6$
λ/2
λ/4
f$_1$
f$_2$
g$_1$
g$_2$
g$_3$
BS
PBS
Camera
Mirrors
Mask
WP
Balanced photodiodes
Probe pulse (λ = 780 nm)
Optical compressor

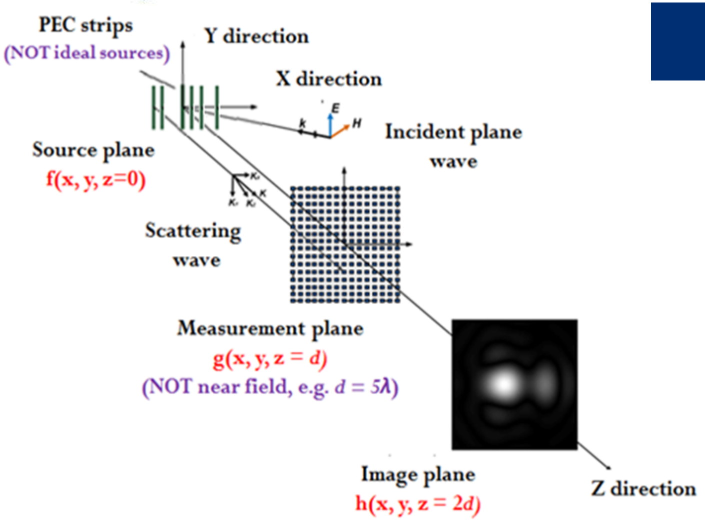
Electro-optic (EO) near-field THz microscope.

* F. Blanchard, A. Doi, T. Tanaka, and K. Tanaka, "Real-time, subwavelength terahertz imaging," Annual Review of Materials Research, vol. 43, pp. 237-259, Mar. 2013.

Challenges:

- ✓ **Sub-wavelength (~λ/10) imaging resolution**
- ✓ **Fresnel region or far-field imaging capabilities**

Introduction

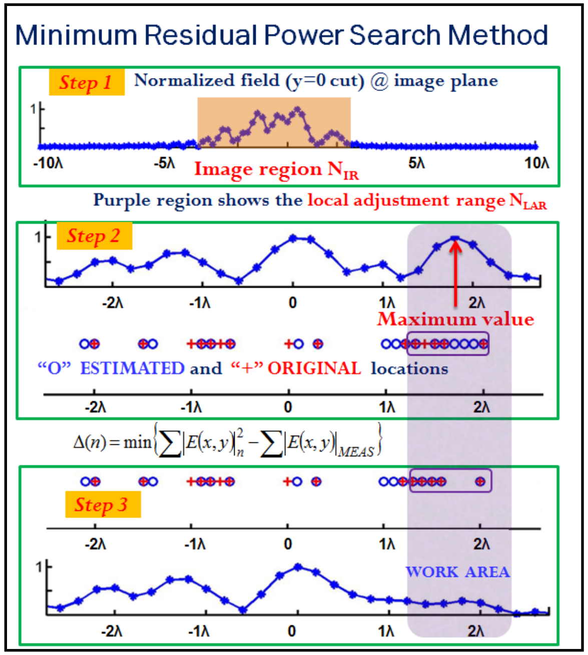

PEC strips
(NOT ideal sources)
Y direction
X direction
E
H
k
Incident plane wave
Source plane
f(x, y, z=0)
Scattering wave
Measurement plane
g(x, y, z = d)
(NOT near field, e.g. d = 5λ)
Image plane
h(x, y, z = 2d)
Z direction

Our previous work:

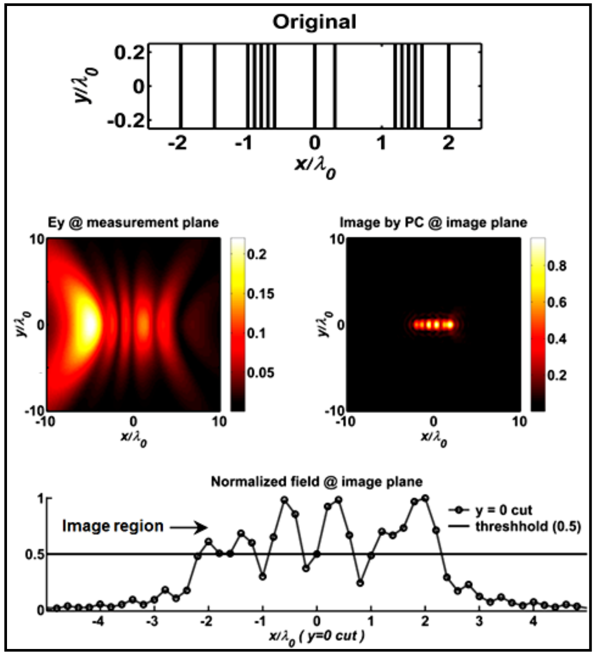
- Sub-wavelength imaging resolution (~λ/10)
- Far-Field imaging (d = 5λ)
- Simple system construction

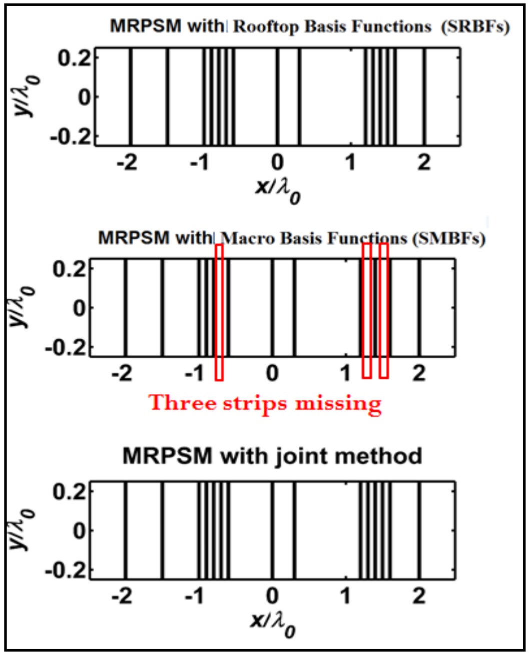
* X. Gu, C. Pelletti, R. Mittra, and Y. Zhang, “Resolution Enhancement of Phase Conjugating Lenses by Using Signal Processing Algorithms,” *IEEE Antennas and Wireless Propagation Letters*, vol. 13, 2014.

* X. Gu, C. Pelletti, R. Mittra and Y. Zhang, “Signal processing approach to electromagnetic sub-wavelength imaging,” AP-S/URSI 2013, Orlando, USA, Jul. 7-13, 2013



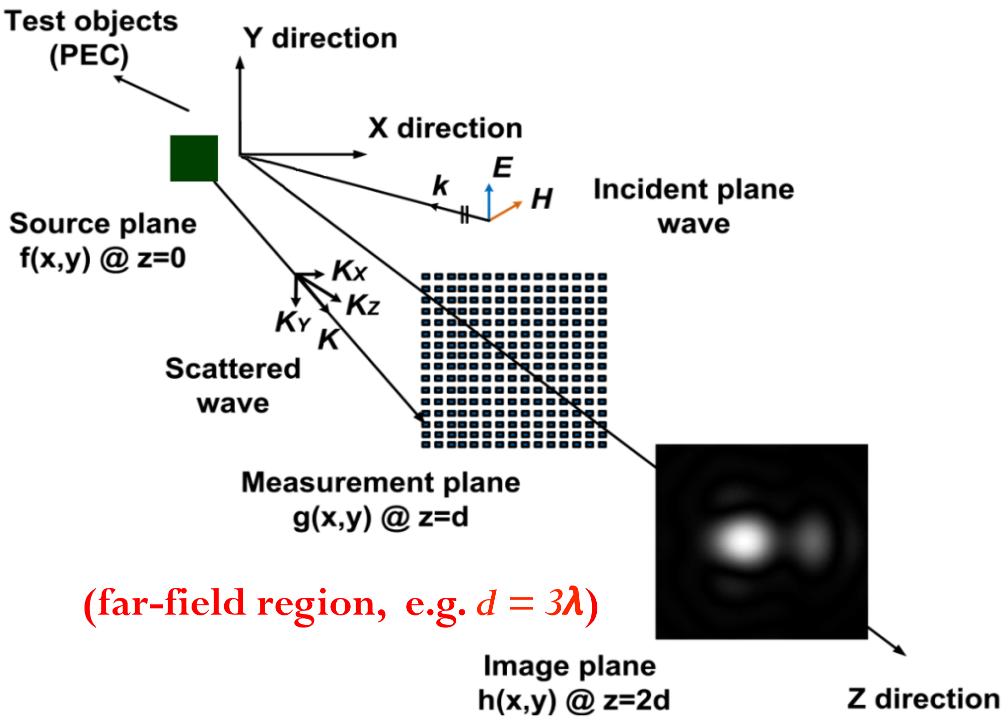
$$\Delta(n) = \min\left\{\sum |E(x,y)|_n^2 - \sum |E(x,y)|_{MEAS}\right\}$$





EM sub-wavelength Imaging Model

Step 1 Electromagnetics (EM) model


Test objects (PEC)

Y direction

X direction

k E H

Incident plane wave

Source plane f(x,y) @ z=0

K_X K_Z K_Y K

Scattered wave

Measurement plane g(x,y) @ z=d

(far-field region, e.g. $d = 3\lambda$)

Image plane h(x,y) @ z=2d

Z direction

- The test objects are assumed to be PEC squares illuminated by a plane wave, that are located in the *x-y* plane at *z=0*.
- The scattered field distribution is measured over a grid, which is set up on the measurement plane at *z=d*.

Step 2

Phase Conjugation (PC)

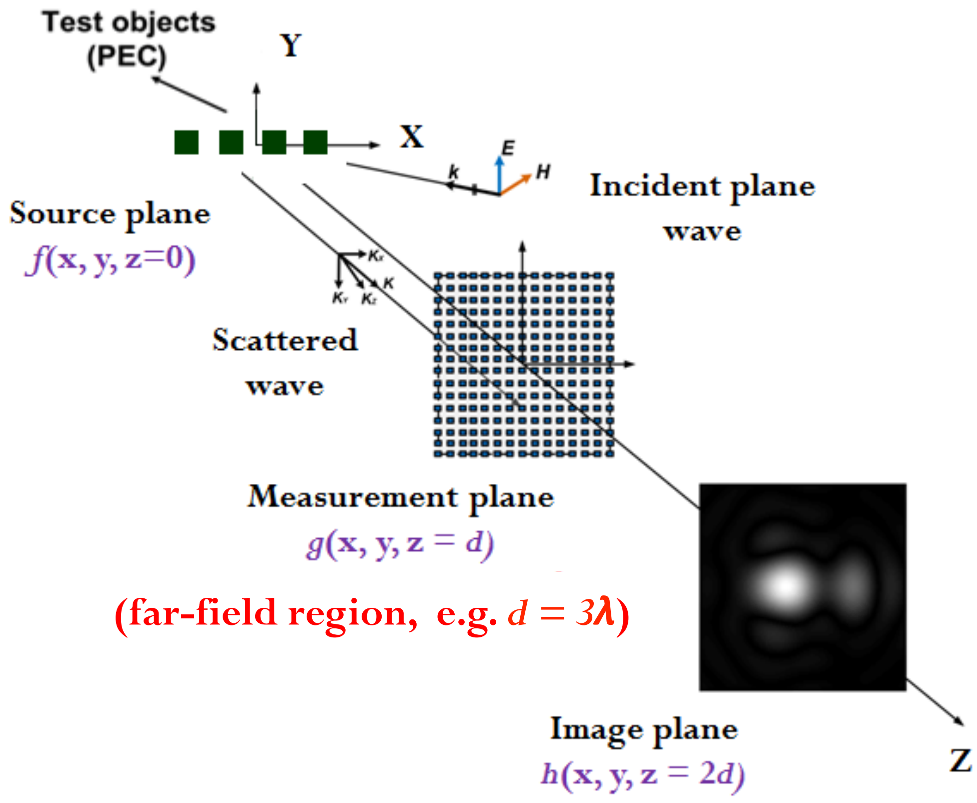
- Algorithms for sub-wavelength imaging via PC are inspired by the principles of imaging lenses.
- The measured data is phase conjugated and propagated to the image plane at *z=2d*, where an image of the original sources is obtained.

Step 3

Signal processing

☐ Basis matrix method with Singular Value Decomposition (SVD)

Phase Conjugation: Basic concepts


Test objects
(PEC)
Y
X
E
H
k
Incident plane wave
Source plane
$f(x, y, z=0)$
K_x
K
K_y
K_z
Scattered wave
Measurement plane
$g(x, y, z=d)$
Image plane
$h(x, y, z=2d)$
Z

(far-field region, e.g. $d = 3\lambda$)

Imaging via PC:

1. Measure the *E*-field radiated from the sources at $z=d$
2. Phase conjugate the spatial field distribution at $z=d$
3. Fourier transform the spatial field distribution to the wavenumber domain at $z=d$
4. Propagate the spectrum to the image plane at $z=2d$
5. Take the inverse transform of the spectral distribution at $z=2d$ to get the desired spatial image

$$g(x,y,z=d)$$

$$\downarrow$$

$$G(k_x,k_y,z=d)=F(k_x,k_y,z=0)e^{-jk_z d}$$

$$H(k_x,k_y,z=2d)=G^*(k_x,k_y,z=d)e^{-jk_z d}$$

$$\downarrow$$

$$H(k_x,k_y,z=2d)=F^*(-k_x,-k_y,z=0)$$

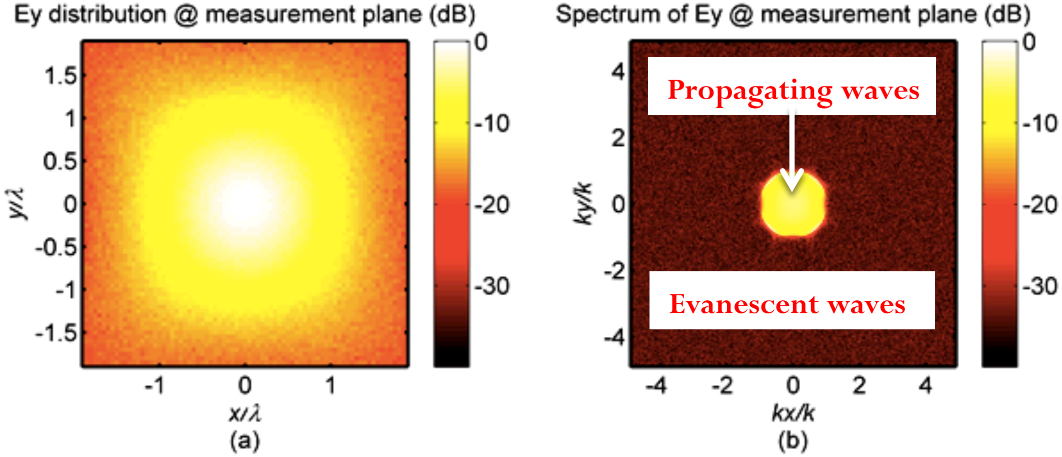
$$\downarrow$$

$$h(x,y,z=2d)=f^*(x,y,z=0)$$

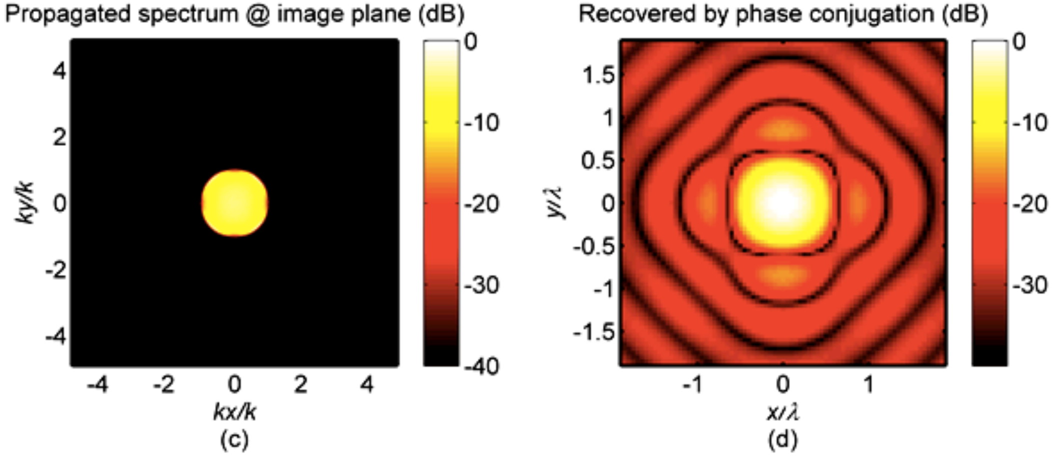
- $f(x, y, z=0)$ and $F(k_x, k_y, z=0)$ represent the spatial and spectral distributions at the source plane
- $g(x, y, z=0)$ and $G(k_x, k_y, z=0)$ represent the spatial and spectral distributions at the measurement plane
- $h(x, y, z=0)$ and $H(k_x, k_y, z=0)$ represent the spatial and spectral distributions at the image plane

Phase Conjugation: Simulations

$g(x, y, z = d)$ $G(k_x, k_y, z = d)$



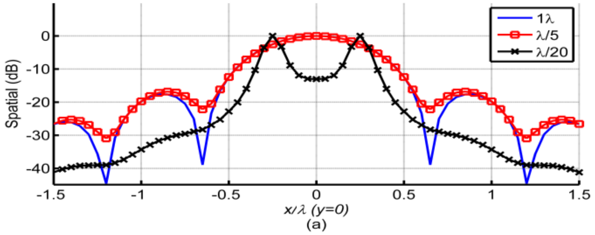
$H(k_x, k_y, z = 2d)$ $h(x, y, z = 2d)$

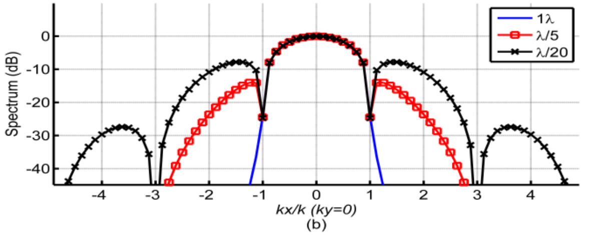


$$k_z = \begin{cases} \sqrt{k_0^2 - (k_x^2 + k_y^2)} & \text{when } \quad k_x^2 + k_y^2 < k_0^2 \quad \text{(Propagating waves)} \\ -j\sqrt{(k_x^2 + k_y^2) - k_0^2} & \text{when } \quad k_x^2 + k_y^2 \geq k_0^2 \quad \text{(Evanescent waves)} \end{cases}$$

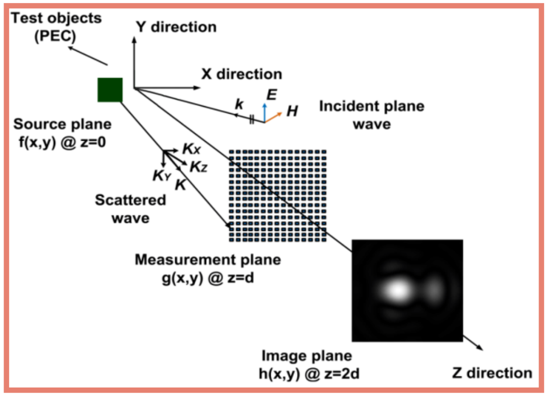
Exponential decay

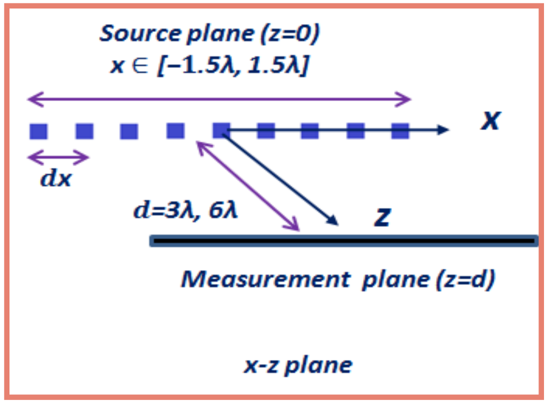
$$\begin{aligned} G(k_x, k_y, z = d) &= F(k_x, k_y, z = 0)\, e^{-jk_z d} \\ &\Downarrow \\ H(k_x, k_y, z = 2d) &= G^*(k_x, k_y, z = d)\, e^{-jk_z d} \\ &\Downarrow \\ H(k_x, k_y, z = 2d) &= F^*(-k_x, -k_y, z = 0) \\ &\Downarrow \\ h(x, y, z = 2d) &= f^*(x, y, z = 0) \end{aligned}$$

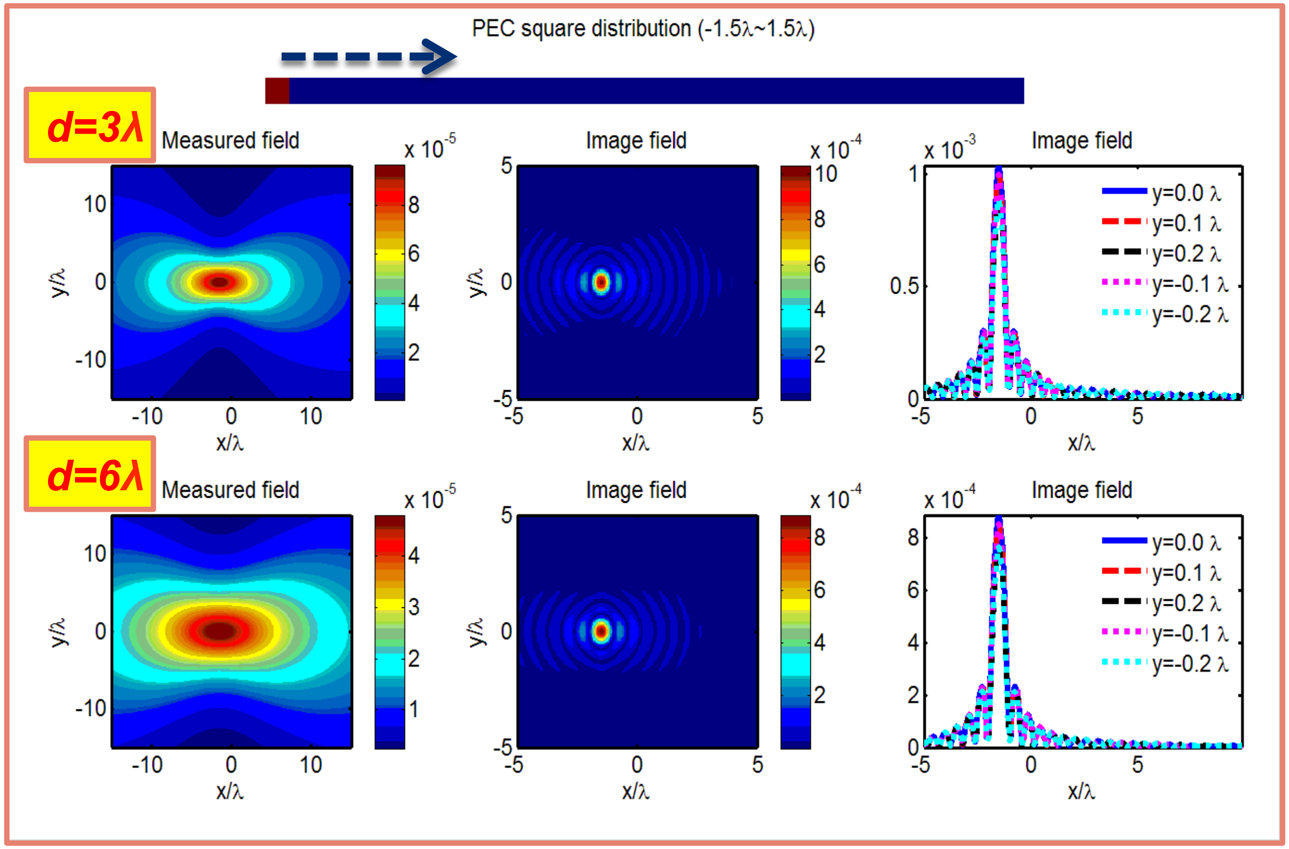




Phase Conjugation: Simulation

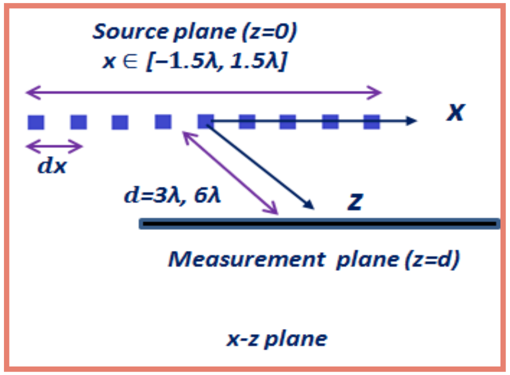

Test objects (PEC)
Y direction
X direction
k
E
H
Incident plane wave
Source plane f(x,y) @ z=0
K_X
K_Z
K_Y
K
Scattered wave
Measurement plane g(x,y) @ z=d
Image plane h(x,y) @ z=2d
Z direction

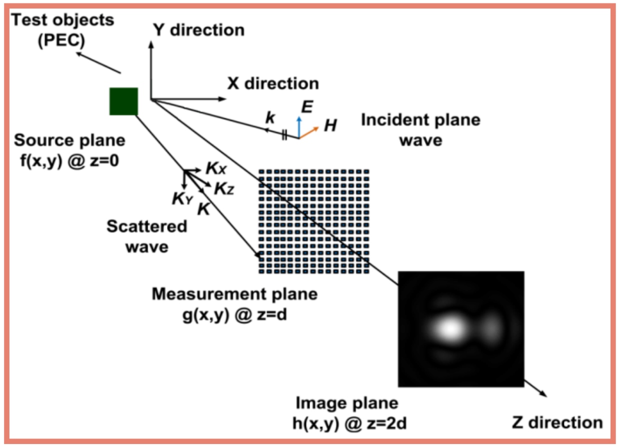

Source plane (z=0)
x ∈ [−1.5λ, 1.5λ]
x
dx
d=3λ, 6λ
z
Measurement plane (z=d)
x-z plane



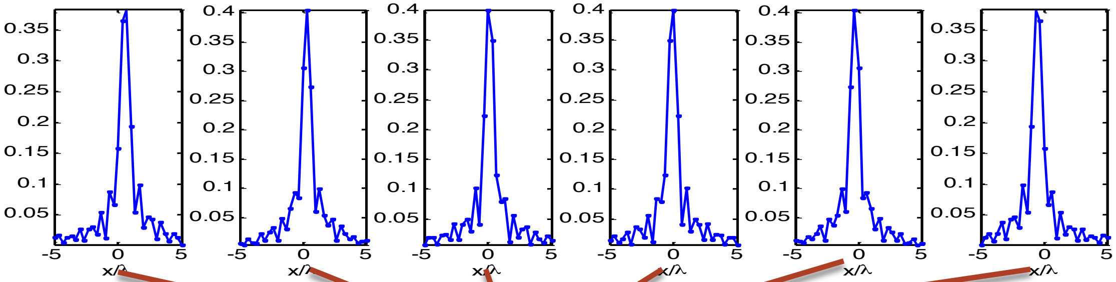
We simulate the case with only one PEC square (0.05λ*0.05λ) one at a time by using FEKO.
The operating frequency is 1GHz.
Normal incident plane wave.
And then, we apply the Phase Conjugation to the measurement data, and extract the image field at the image plane.

Basis matrix method with Singular Value Decomposition (SVD)


Source plane (z=0)
x ∈ [−1.5λ, 1.5λ]
x
dx
d=3λ, 6λ
z
Measurement plane (z=d)
x-z plane


Test objects (PEC)
Y direction
X direction
k
E
H
Incident plane wave
Source plane f(x,y) @ z=0
Kx
Kz
KY
K
Scattered wave
Measurement plane g(x,y) @ z=d
Image plane h(x,y) @ z=2d
Z direction

Phase Conjugate the measurement data, and extract the image field (e.g. y=0 cut) at the image plane as a vector of the basis matrix.

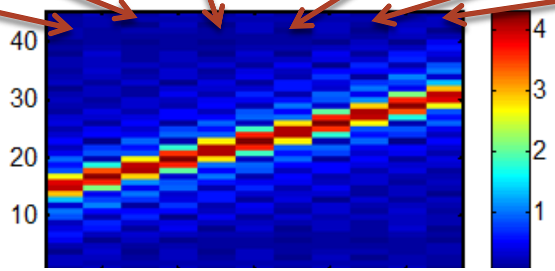


Observations at the image plane along x direction

$$b_{(N\times1)} = A_{(N\times M)}\, x_{(M\times1)}$$

Observations (N) at the image plane along x direction for each PEC square

Basis matrix



-1.5λ -0.9λ -0.3λ 0.3λ 0.9λ 1.5λ

Different positions (M) of PEC square

Concept of SVD based algorithm

$$\boldsymbol{b}_{(N\times1)} = \boldsymbol{A}_{(N\times M)}\, \boldsymbol{x}_{(M\times1)}$$

ill-conditioned

Singular Value Decomposition (SVD)

$$\boldsymbol{A}_{(N\times M)} = \boldsymbol{U}_{(N\times N)}\, \boldsymbol{S}_{(N\times M)}\, \boldsymbol{V}_{(M\times M)}$$

$$\boldsymbol{S}_{(N\times M)} = \left[\begin{array}{ccccc} \lambda_1 & 0 & \dots & 0 & 0 \\ 0 & \lambda_2 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \lambda_K & \dots & 0 \\ \hline \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \lambda_N \\ 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \end{array}\right]$$

Threshold i.e. 0.001

$$\boldsymbol{U^s}_{(N\times K)} = [\mathbf{u}^1{}_{(N\times1)}, \mathbf{u}^2{}_{(N\times1)}, \dots, \mathbf{u}^K{}_{(N\times1)}]$$

Projection (dot product)

$$\boldsymbol{A}_{(N\times M)} = [\mathbf{a}^1{}_{(N\times1)}, \mathbf{a}^2{}_{(N\times1)}, \dots, \mathbf{a}^M{}_{(N\times1)}]$$

$$\boldsymbol{b}_{(N\times1)}$$

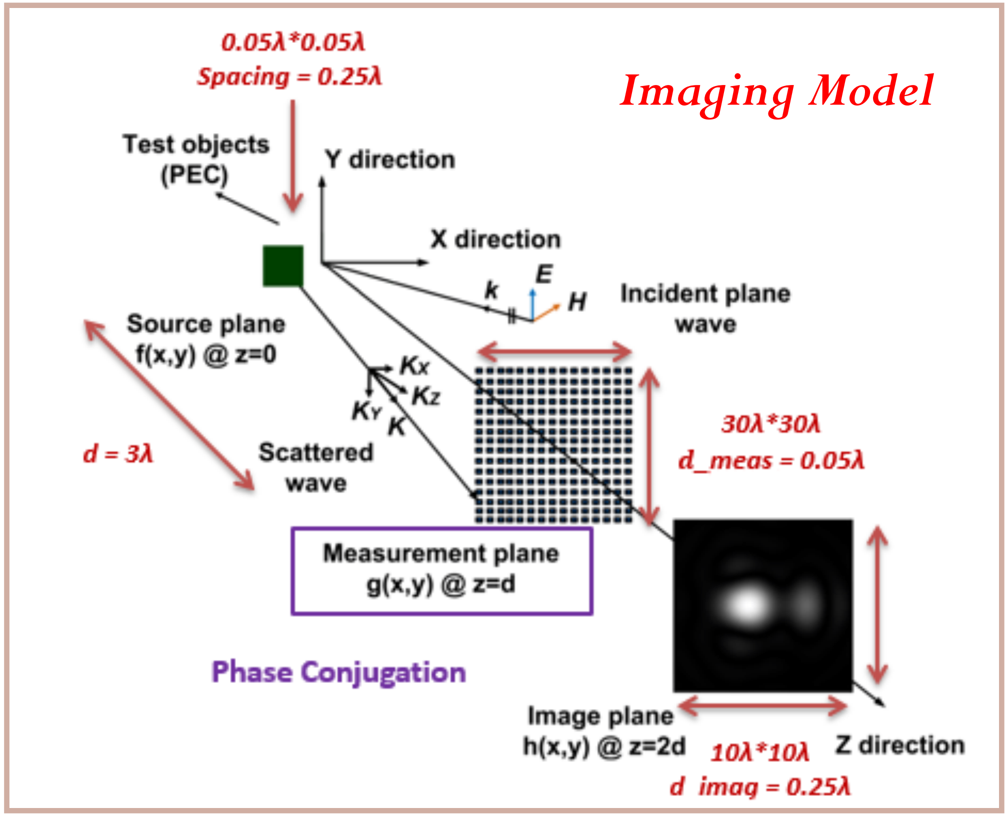
$$\boldsymbol{A^n}_{(K\times M)} = [\mathbf{a}^{n1}{}_{(K\times1)}, \mathbf{a}^{n2}{}_{(K\times1)}, \dots, \mathbf{a}^{nM}{}_{(K\times1)}]$$

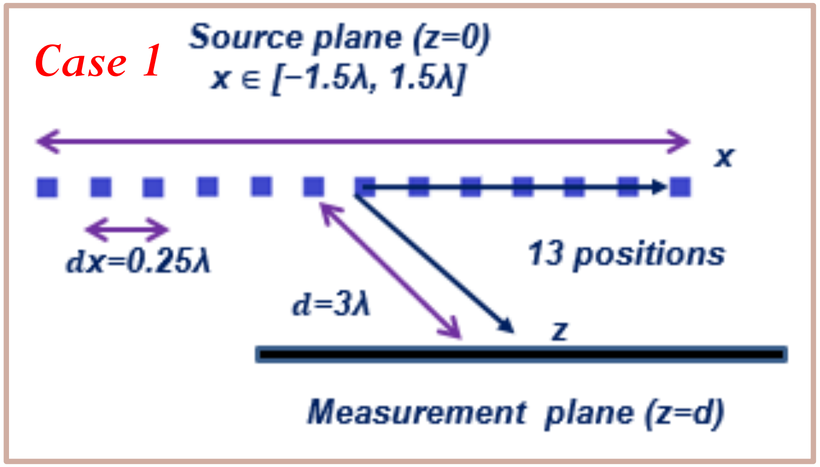
$$\boldsymbol{b^n}_{(K\times1)}$$

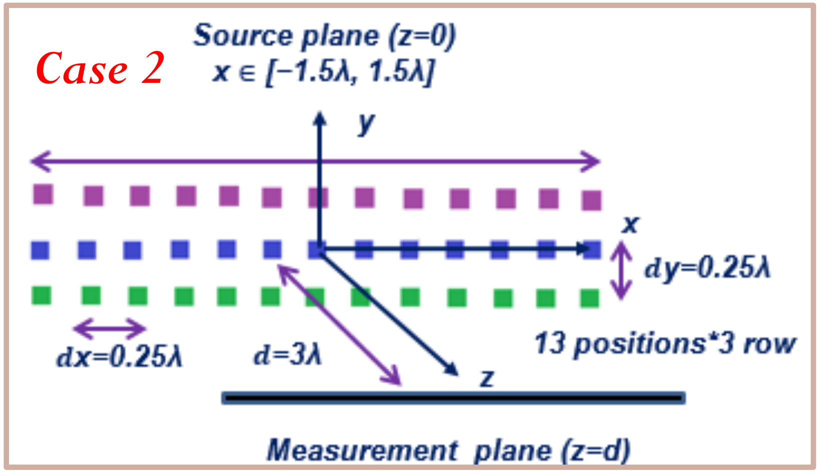
$$\boldsymbol{b^n}_{(K\times1)} = \boldsymbol{A^n}_{(K\times M)}\, \boldsymbol{x}_{(M\times1)}$$

well-conditioned

Numerical Considerations


Imaging Model
0.05λ*0.05λ
Spacing = 0.25λ
Test objects
(PEC)
Y direction
X direction
k
E
H
Incident plane
wave
Source plane
f(x,y) @ z=0
Kx
Kz
Ky
K
d = 3λ
Scattered
wave
30λ*30λ
d_meas = 0.05λ
Measurement plane
g(x,y) @ z=d
Phase Conjugation
Image plane
h(x,y) @ z=2d
10λ*10λ
d_imag = 0.25λ
Z direction


Case 1
Source plane (z=0)
x ∈ [−1.5λ, 1.5λ]
x
dx=0.25λ
13 positions
d=3λ
z
Measurement plane (z=d)

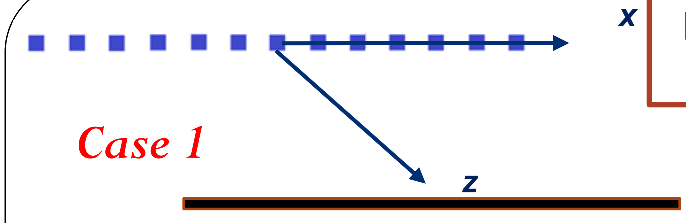

Case 2
Source plane (z=0)
x ∈ [−1.5λ, 1.5λ]
y
x
dy=0.25λ
13 positions*3 row
dx=0.25λ
d=3λ
z
Measurement plane (z=d)

Numerical Results: x-direction Without noise

Black starts are original positions of PEC squares

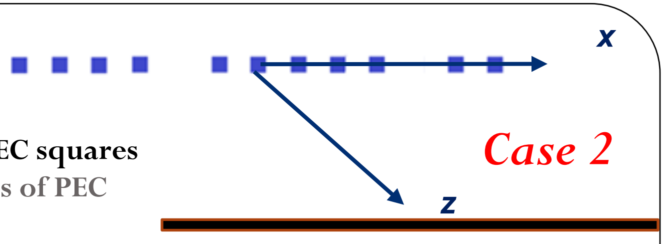
Blue bars are the recovered positions of PEC squares

Red line is the threshold of 0.5 (normalized)



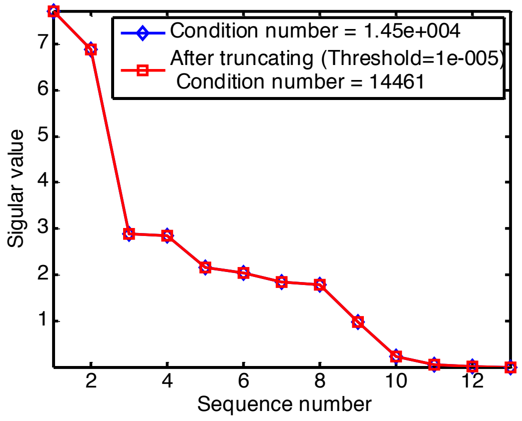
Case 1

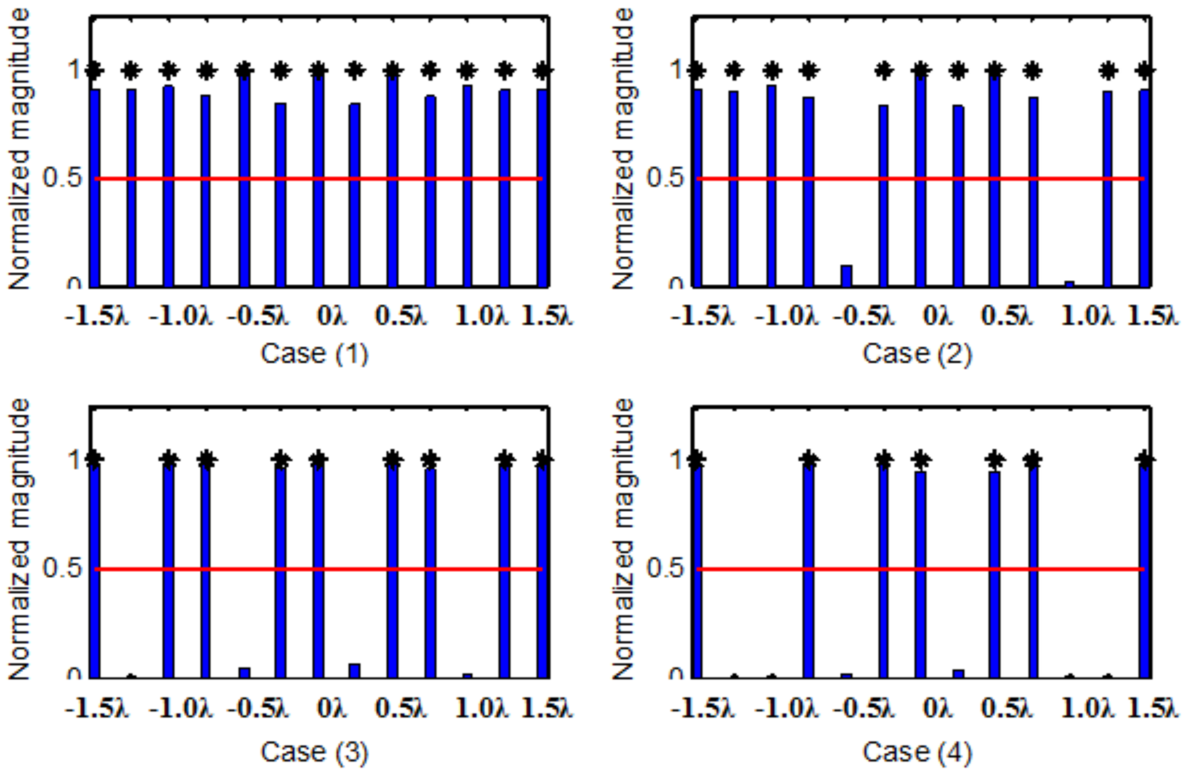
Measurement plane (z=d)

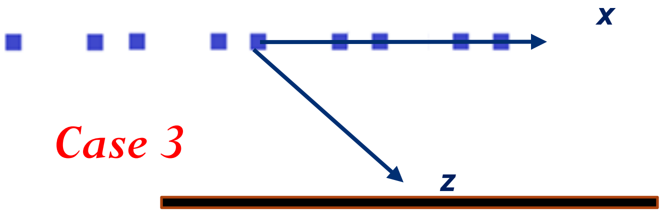


Case 2

Measurement plane (z=d)

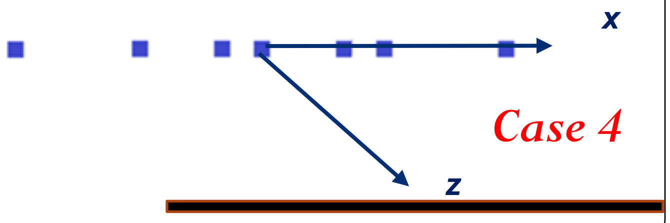






Case 3

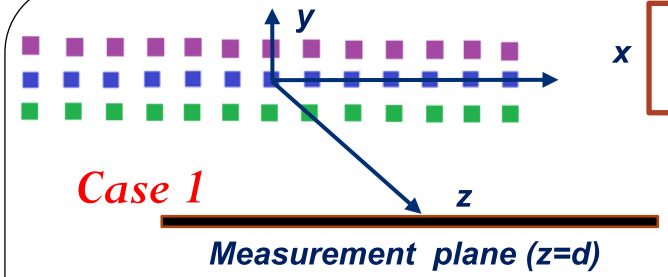
Measurement plane (z=d)

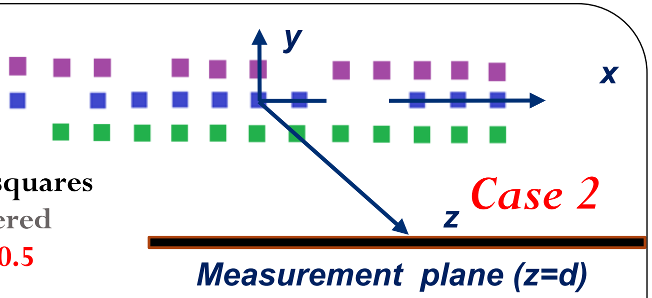


Case 4

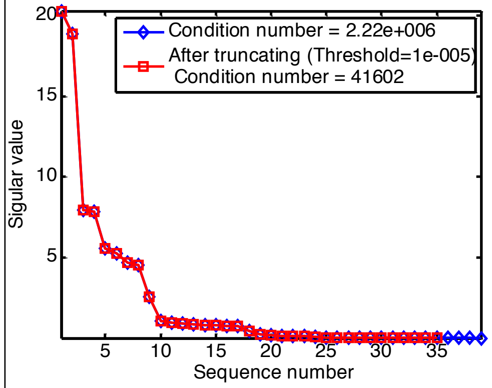
Measurement plane (z=d)

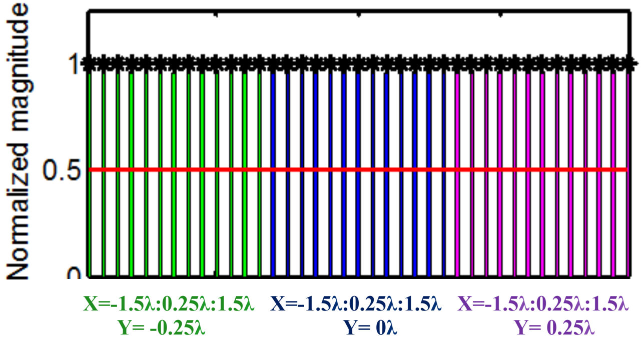
Numerical Results: x-y plane Without noise


y
x
z
Case 1
Measurement plane (z=d)

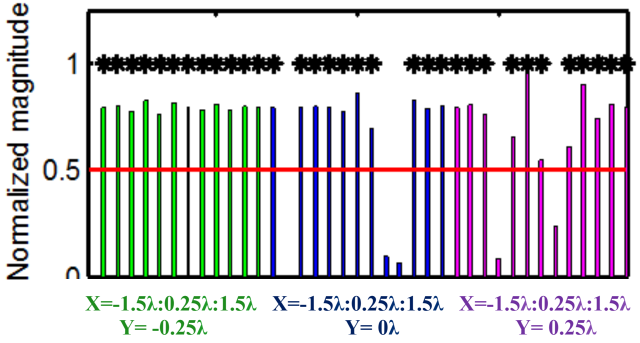

y
x
z
Case 2
Measurement plane (z=d)

Black starts are original positions of PEC squares Green/blue/purple bars are the recovered positions Red line is the threshold of 0.5 (normalized)

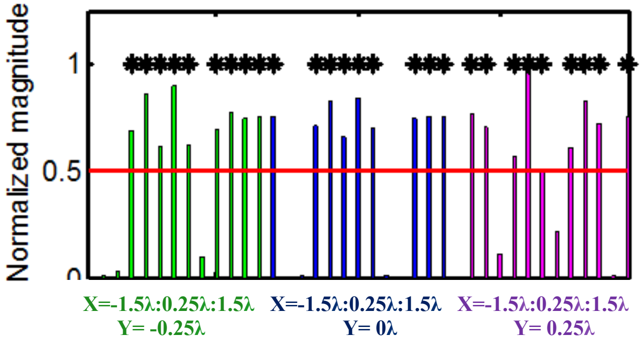




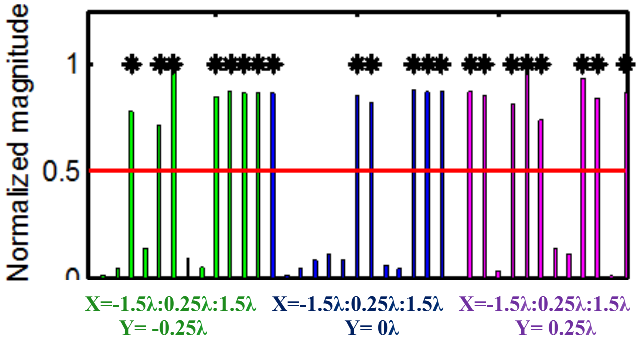
Case (1)



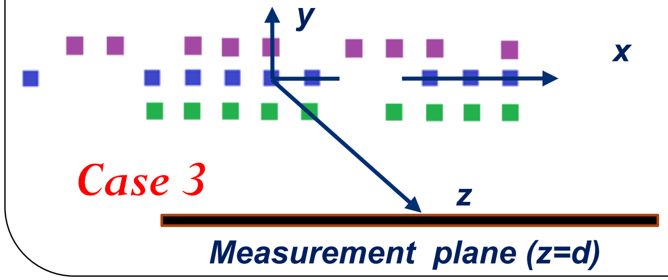
Case (2)

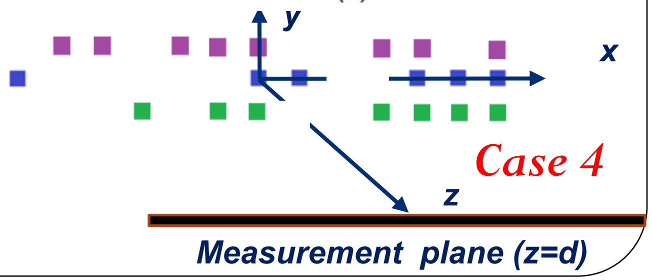


Case (3)

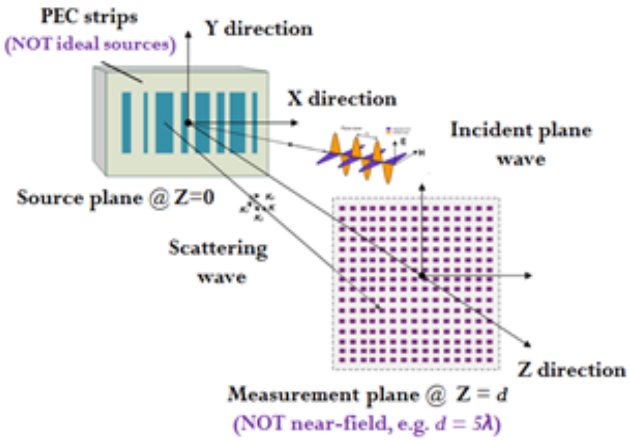


Case (4)


y
x
z
Case 3
Measurement plane (z=d)


y
x
z
Case 4
Measurement plane (z=d)

Conclusions

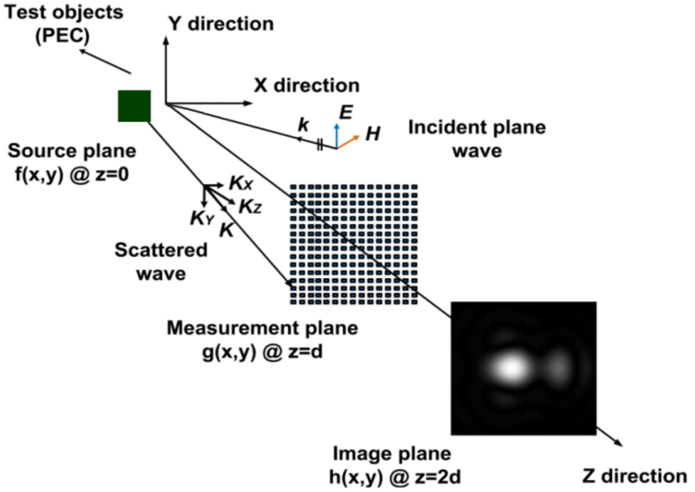

PEC strips
(NOT ideal sources)
Y direction
X direction
Incident plane wave
Source plane @ Z=0
Scattering wave
Z direction
Measurement plane @ Z = d
(NOT near-field, e.g. d = 5λ)

Our previous work:

- Sub-wavelength imaging resolution (~λ/10)
- Far-Field imaging ($d = 5\lambda$)
- Simple system construction

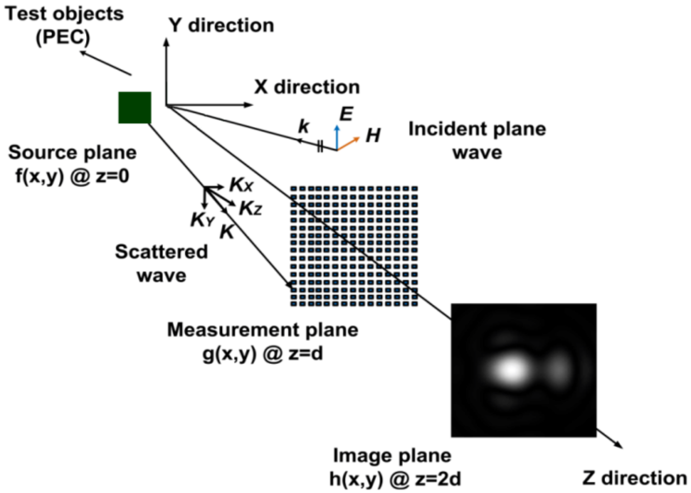
* X. Gu, C. Pelletti, R. Mittra, and Y. Zhang, "Resolution Enhancement of Phase Conjugating Lenses by Using Signal Processing Algorithms," *IEEE Antennas and Wireless Propagation Letters*, vol. 13, 2014.

* X. Gu, C. Pelletti, R. Mittra and Y. Zhang, "Signal processing approach to electromagnetic sub-wavelength imaging," AP-S/URSI 2013, Orlando, USA, Jul. 7-13, 2013

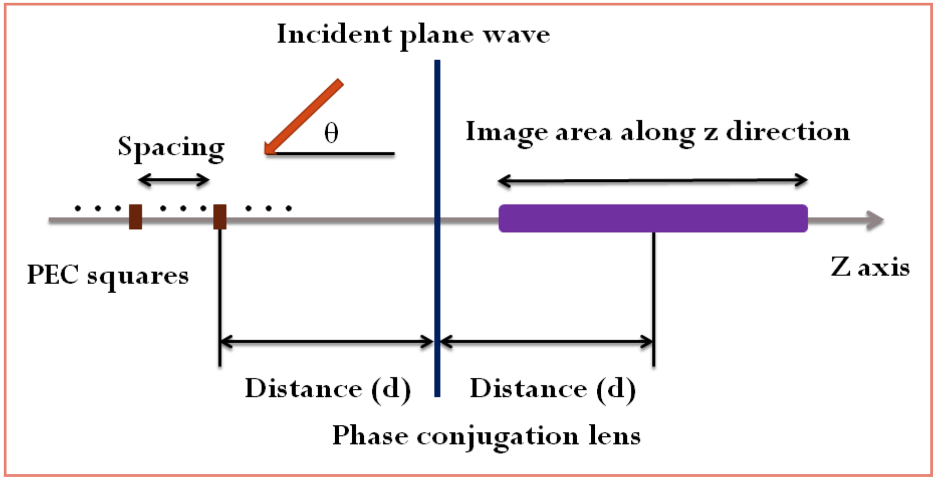

Test objects (PEC)
Y direction
X direction
k
E
H
Incident plane wave
Source plane f(x,y) @ z=0
Kx
Kz
Ky
K
Scattered wave
Measurement plane g(x,y) @ z=d
Image plane h(x,y) @ z=2d
Z direction

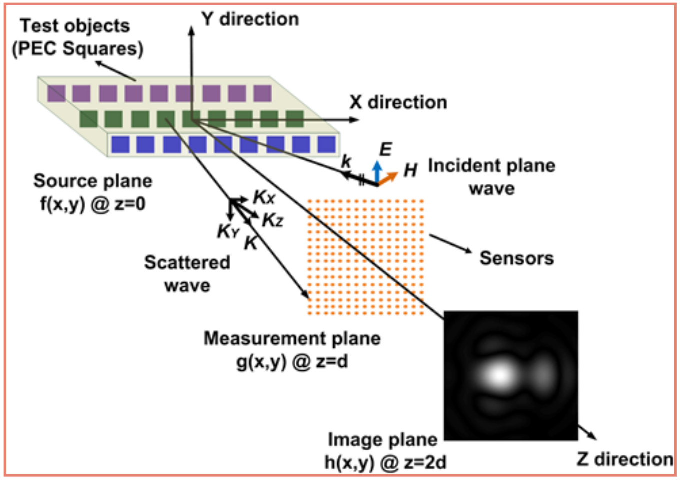
- 2D sub-wavelength imaging (resolution ~λ/4)
- Fresnel region or far-field imaging (d = 3λ/6λ)
- Simple system construction
- Easy to be extended for 3D distribution

Future work


Test objects (PEC)
Y direction
X direction
E
k
H
Incident plane wave
Source plane f(x,y) @ z=0
K_X
K_Z
K_Y
K
Scattered wave
Measurement plane g(x,y) @ z=d
Image plane h(x,y) @ z=2d
Z direction

- *Improved resolution*
- *Dielectric targets*
- *Longitudinal direction sub-wavelength imaging*
- *3D sub-wavelength imaging problem*


Incident plane wave
θ
Spacing
Image area along z direction
PEC squares
Z axis
Distance (d)
Distance (d)
Phase conjugation lens


Test objects (PEC Squares)
Y direction
X direction
E
k
H
Incident plane wave
Source plane f(x,y) @ z=0
K_X
K_Z
K_Y
K
Sensors
Scattered wave
Measurement plane g(x,y) @ z=d
Image plane h(x,y) @ z=2d
Z direction

THANK YOU

how
which
why
how
when
what
where
which